Hubble's law and the expanding sphere

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Abstract

Considerable progress has been made in determining the uniform, isotropic expansion (**Hubble flow**) of an isolated sphere of radius R and mass M over the past decade. In this article we take a look at Hubble's law and outline the cosmological context of equations that describe the time evolution of an expanding sphere.

Edwin Powell Hubble was an American astronomer who played a crucial role in establishing the fields of extragalactic astronomy and observational cosmology. He proved that many objects previously thought to be clouds of dust and gas and classified as "nebulae" were actually galaxies beyond the Milky Way.

Gravitational binding energy

A gravitational binding energy is the minimum energy that must be added to a system for the system to cease being in a gravitationally bound state. For a spherical mass of uniform density, the gravitational binding energy U is given by the formula:

$$U = -\frac{3GM^2}{5R}$$

where G is the gravitational constant, M is the mass of the sphere, and R is its radius.

$$\frac{U}{Mc^2} = -0.3 \times \frac{R_s}{R}$$

where $R_s = \frac{2GM}{c^2}$ is the Schwarzschild radius of the sphere.

If $R = R_s$:

$$U = -\,0.3Mc^2$$

$$\boxed{M_{binding} = -\,0.3M}$$

Hubble's Law

Back in 1700s, people thought the stars of our galaxy structured the universe, that the galaxy was nearly static, and that the universe was essentially unexpanding with neither a beginning nor an end to time. A situation marked by difficulty with the idea of a static and unchanging universe, was that according to the Newtonian theory of gravitation, each star in the universe supposed to be pulled towards every other star with a force that was weaker the less massive the stars and farther they were to each other. It was this force caused all the stars fall together at some point. So how could they remain static? Wouldn't they all collapse in on themselves? A balance of the predominant attractive effect of the stars in the universe was required to keep them at a constant distance from each other. Einstein was aware of this problem. He introduced a term so-called cosmological constant in order to hold a static universe in which gravity is a predominant attractive force. This had an effect of a repulsive force, which could balance the predominant attractive force. In this way it was possible to allow a static cosmic solution. Enter the American astronomer Edwin Hubble. In 1920s he began to make observations with the hundred inch telescope on Mount Wilson and through detailed measurements of the spectra of stars he found something most peculiar: stars moving away from each other had their spectra shifted toward the red end of the spectrum in proportion to the distance between them (This was a Doppler effect of light: Waves of any sort − sound waves, light waves, water waves − emitted at some frequency by a moving object are perceived at a different frequency by a stationary observer. The resulting shift in the spectrum will be towards its red part when the source is moving away and towards the blue part when the source is getting closer). And he also observed that stars were not

uniformly distributed throughout space, but were gathered together in vast collections called galaxies and nearly all the galaxies were moving away from us with recessional velocities that were roughly dependent on their distance from us. He reinforced his argument with the formulation of his well known Hubble's law. The rate of expansion $\frac{dR}{dt}$ and radius **R** of the expanding sphere is now related by Hubble's Law:

$$v = \frac{dR}{dt} = HR$$

where H is the Hubble parameter which is a value that is time dependent. Since the volume of a spherical body is assumed to be V = $\frac{4\pi R^3}{3}$:

$$\frac{dV}{dt} = 3HV$$

If the sphere is expanding adiabatically then it will satisfy the first law of thermodynamics:

$$0 = dQ = dU + PdV$$

where Q is the total heat which is assumed to be constant, U is the internal energy of the matter in the sphere and P is the pressure.

$$-\frac{dU}{dt} = P\frac{dV}{dt}$$

$$-\frac{dU}{dt} = 3HPV$$

Mathematically:

$$P = \frac{F}{A}$$

where P is the pressure, F is the force and A is the surface area of a sphere.

$$-\frac{dU}{dt} = 3FH\frac{V}{A}$$

Since $A = 4\pi R^2$ and $V = \frac{4\pi R^3}{3}$:

$$-\frac{dU}{dt} = FHR$$

$$\boxed{-\frac{dU}{dt} = F \times v}$$

Density Parameter

The density parameter, Ω, is defined as the ratio of the actual (or observed) density $\rho = \frac{M}{V}$ to the critical density $\boldsymbol{\rho_c}$ of the expanding sphere.

We define a critical density

$$\rho_c = \frac{3H^2}{8\pi G}$$

and the density parameter

$$\Omega = \frac{8\pi G\rho}{3H^2} = \frac{8\pi GM}{3VH^2}$$

$$\Omega = \frac{2GM}{H^2R^2} \times \frac{1}{R}$$

$$\boxed{v = \sqrt{\frac{2GM}{R\Omega}}}$$

$$-\frac{dU}{dt} = F\sqrt{\frac{2GM}{R\Omega}}$$

Deceleration parameter

The deceleration parameter, $\mathbf{q_0}$, indicates the rate at which the expansion of sphere is slowing due to self-gravitation. The time derivative of the Hubble parameter $\frac{dH}{dt}$ can be written in terms of the deceleration parameter $\mathbf{q_0}$:

$$\frac{dH}{dt} = -H^2(1 + q_0)$$

Since $H = \frac{v}{R}$:

$$\frac{d}{dt}\left(\frac{v}{R}\right) = -H^2(1 + q_0)$$

$$\frac{\frac{dv}{dt}R - v\frac{dR}{dt}}{R^2} = -H^2(1 + q_0)$$

$$\frac{dv}{dt}R - v\frac{dR}{dt} = -H^2R^2(1 + q_0)$$

$$\frac{dv}{dt}R - v^2 = -v^2(1 + q_0)$$

The acceleration of expansion of the sphere, $a = \frac{dv}{dt} = -vHq_0$

$$a = -Hq_0\sqrt{\frac{2GM}{R\Omega}}$$

$$\boxed{q_0 = -\frac{a}{H\sqrt{\frac{2GM}{R\Omega}}}}$$

Thermodynamics

1. Energy is conserved
2. Randomness increases
3. Absolute Zero temperature is Unattainable

Because:

$$\boxed{T_{BH} \propto \frac{1}{M}}$$

- Tiny Black Hole is hot
- Big Black Hole is cold

60% Dark Energy (we don't know what it is)

35% Cold dark matter (we don't know what it is)

5% Nuclei and electrons (visible as stars ~0.5%)

Objects that travel with v << c obey this relation:

$$\boxed{z = \frac{\Delta\lambda}{\lambda} = \frac{v}{c}}$$

- Objects moving away from observer → frequency decreases → wavelength increases (**red shift**)
- Objects moving towards observer → frequency increases → wavelength decreases (**blue shift**)

Einstein Theory → 4 dimensions (length, width, depth, and time)

String theory → 4 dimensions + 7 other dimensions

11^{th} dimension holds the universe together

The black hole no hair theorem:

Mass, charge, and angular momentum are the only properties a black hole can possess

The Sky is Dark at Night → there must be some limit to the observable Universe.

Density parameter (Ω) = $\frac{\text{density}}{\text{critical density}}$

- $\Omega > 1 \rightarrow$ closed universe
- $\Omega = 1 \rightarrow$ flat universe
- $\Omega < 1 \rightarrow$ open universe

Gamma radiation $\rightarrow \lambda < 0.001$ nm, $E_{\gamma} > 1.24$ MeV

$\downarrow$

Produced in nuclear reactions and other very high energy processes

X-rays $\rightarrow 0.001$ nm $< \lambda < 10$ nm, 124 eV $< E_{x\text{-}ray} < 1.24$ MeV

$\downarrow$

Produced in supernovae remnants and the solar corona, as well as in the hot gas between galaxy clusters

Thomson Scattering ($h\upsilon \ll m_0c^2$**)**

The photon and electron just both bounce off each other, changing their direction, but there is no exchange of energy

Compton scattering ($h\upsilon > m_0c^2$**)**

A photon of high energy collides with a stationary electron and transfers part of its energy and momentum to the electron, decreasing its frequency in the process

The maximum energy gained by photons via **inverse Compton scattering** is proportional to its initial energy multiplied by the square of twice the Lorentz factor (where the **Lorentz factor** is given by $\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$ and v is the velocity of the electron):

$$E_{max} = (h\upsilon)_{max} \propto 4\,\gamma^2\, h\upsilon_0$$

- 1 eV = 1.602×10^{-19} J = 1.602×10^{-12} erg

Brown dwarf

- Too big to be a planet
- Too small to be a star

Pulsars → Rotating neutron stars emitting beams of particles and electromagnetic radiation

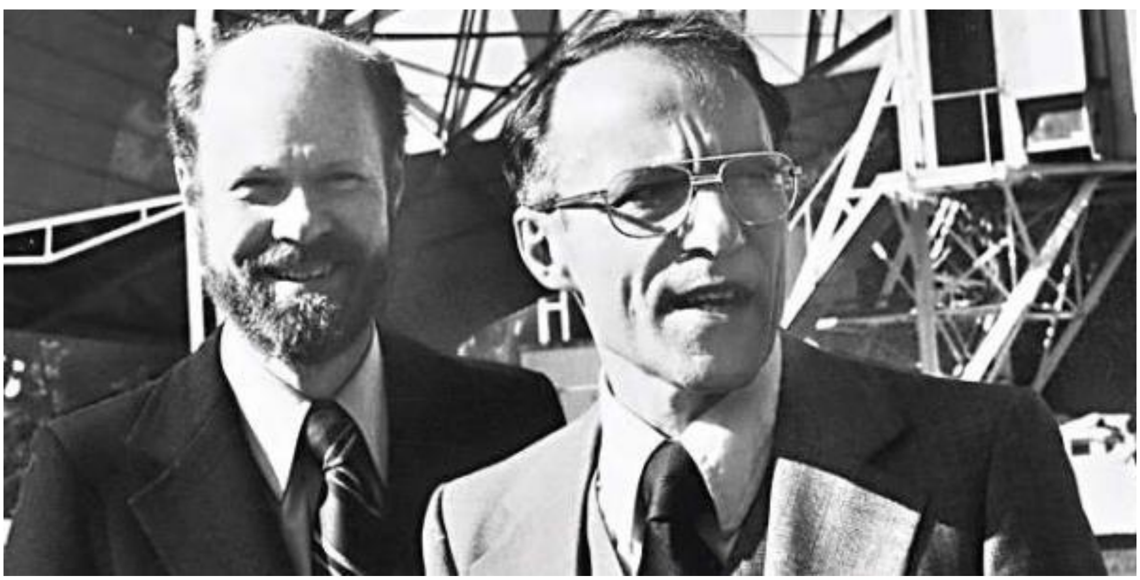
- **Gravity** warps space and time
- Even photons with no mass can have their trajectories bent → **gravitational lensing**

$\lambda \ll R$	$\sigma \approx \pi R^2$	**Geometrical scattering**
$\lambda \approx R$	$\sigma \propto \frac{1}{\lambda}$	**Mie scattering**
$\lambda \gg R$	$\sigma \propto \frac{1}{\lambda^4}$	**Rayleigh scattering**

$$\text{Energy of radiation} \propto \frac{1}{(\text{scale factor of the universe})}$$

$$\text{Energy density of radiation} \propto \frac{1}{(\text{scale factor of the universe})^4}$$

$$\text{Relative brightness} = \frac{\text{absolute brightness}}{(\text{distance})^2}$$



In 1964, **Arno Penzias and Robert Wilson**, two engineers at Bell Labs in New Jersey discovered Cosmic Background radiation when trying to get rid of noise from an antenna aimed at telecommunications satellites.

$$\frac{d(\text{photons})}{dt} = \text{Metric} + \text{Compton Scattering}$$

$$\frac{d(\text{electrons+hadrons})}{dt} = \text{Metric} + \text{Compton Scattering} + \text{Weak Interaction}$$

$$\frac{d(\text{neutrinos})}{dt} = \text{Metric} + \text{Weak Interaction}$$

Spacetime tells matter how to move, matter tells spacetime how to curve

Special Relativity:

- The speed of light is the same for any observer

Universe expands as time passes

Universe cools down as time passes

- If A couples to B, and B to C, A should couple to C.

Spacetime curvature ⟵ $G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$ ⟶ **Energy-momentum density of matter**

Gravitation = Geometry of space-time

At scale L ~ $\sqrt{\frac{G\hbar}{c^3}}$, energy fluctuations become so large that even spacetime geometry is no longer smooth at all.

3 types of geometries for our universe:

- Hyperbolic (negative curvature)
- Elliptic (positive curvature)
- Euclidean (zero curvature)

Photon + Hydrogen atom → proton + electron (**photodissociation**)

Proton + electron → Photon + Hydrogen atom (**radiative recombination**)

General theory of relativity $\left\{ \right.$ Equivalence between inertial mass and gravitational mass

Big Bang	Birth of the Universe
Planck Era	String Theory / Quantum Cosmology
Inflation Era	Symmetry Breaking → Exponential Expansion
Quark Era	Free Quarks in Thermal Equilibrium
Hadron Era	Matter Anti Matter Asymmetry
Lepton Era	Rapid Expansion/cooling (leptons/photons equilibrium)

Radiation Era	Nucleosynthesis, Decoupling
Matter Era	Structure Formation, first galaxies
Acceleration Era	Acceleration phase of the Universe

Newton Theory:

Weight is proportional to Mass

Einstein Theory:

Energy is proportional to Mass

Neither explained origin of Mass

- **Electroweak theory** predicted a heavy version of the photon called the Z^0 which was discovered in 1983.
- **Quantum field theory** which postulates that matter is composed out of elementary particles bound together by forces, mediated by exchange of other elementary particles.

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